EFFECT OF PROXIMITY OF BARRIER ON LIFTING FORCE PRODUCED BY VERTICAL SOLID JETS

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The effect of proximity to the ground on the lifting force generated by a vertical solid jet is studied in connection with development of vertical takeoff and landing devices and of air cushion devices. Such a study was made in [1] for planar flow by an incompressible ideal fluid. There a generalization of the results obtained on a compressible fluid was made by the approximation method. In the present work the planar problem of streamline flow past a dihedral barrier of a gas jet emerging from a channel with parallel walls was solved by the Chaplygin–Fal'kovich method [2, 3]. The results of [1, 4-9] follow as a particular case from the solution obtained. Calculations were carried out clarifying the effect of the proximity of a barrier and the lifting effect of a fluid on flow characteristics at subsonic speeds.

1. The subsonic gas jet, contained by semiinfinite parallel walls the distance between which is 2d, extends upward infinitely with velocity v_1 and density ρ_1. At some distance s from the outlet of the channel the jet encounters a dihedral barrier in its path which splits it into two symmetrical branches of width δ at infinity. In view of the symmetry of the problem it is sufficient to examine only one half of the jet (Fig. 1).

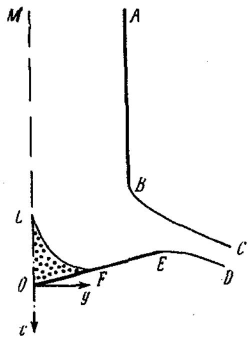

Fig. 1

Here AB is the channel wall; MLO is the axis of symmetry; OFE is the face of the barrier, of length l ; BC and ED are the free surfaces of the stream at which the gas velocity $v_2 > v_1$, of density ρ_2; m is the angle between the stream and the x axis at infinity; s is the distance of the barrier apex 0 from the channel outlet; and $\alpha = \sigma\pi$ $(0 < \sigma < 1)$ is the angle which the surface OFE forms with the direction of the velocity v_1.

It is assumed that a dead air zone LOF is formed at the surface of the barrier with a curved surface LF at which gas velocity $v_0 > v_1$ and the density is ρ_0. The pattern of laminar jet flow with a dead space in front of it was suggested by S. A. Chaplygin [8].

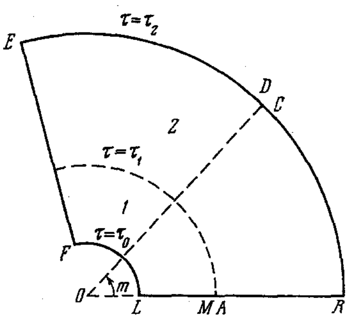

Fig. 2

We define the flow rate of gas in the channel as Q and assume that along the flow line MLFED the flow function $\psi = 0$; consequently along the line ABC, $\psi = Q/2$.

At the surface of the velocity trajectory in polar coordinates τ, θ $(\tau = v^2/v_m^2)$, where v is the velocity and v_m the maximum velocity, and θ is the angle between the velocity and the x axis (Fig. 1), the entire region occupied by the current will correspond to a sector of a ring with a cross section along the ray $\theta = 0$ and radii τ_0 and τ_2, and aperture angle α (Fig. 2). The values which the current function should take along the boundaries of the velocity trajectory region are as follows:

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$$\begin{aligned}
&\psi = 0 \quad \text{at} \quad \theta = 0, \quad \tau_0 \leqslant \tau \leqslant \tau_1 \\
&\psi = 0 \quad \text{at} \quad \theta = \alpha, \quad \tau_0 \leqslant \tau \leqslant \tau_2 \\
&\psi = Q/2 \quad \text{at} \quad \theta = 0, \quad \tau_1 \leqslant \tau \leqslant \tau_2
\end{aligned} \tag{1.1}$$

$$\begin{aligned}
&\psi = 0 \quad \text{at} \quad \tau = \tau_0, \quad 0 \leqslant \theta \leqslant \alpha \\
&\psi = 0 \quad \text{at} \quad \tau = \tau_2, \quad m \leqslant \theta \leqslant \alpha \\
&\psi = Q/2 \quad \text{at} \quad \tau = \tau_2, \quad 0 \leqslant \theta \leqslant m
\end{aligned} \tag{1.2}$$

Thus, the solution of the given problem comes down to Dirichlet's internal solution of the problem for Chaplygin's equation

$$4\tau^2(1-\tau)\frac{\partial^2\psi}{\partial\tau^2} + 4\tau[1+(\beta-1)\tau]\frac{\partial\psi}{\partial\tau} + [1-(2\beta+1)\tau]\frac{\partial^2\psi}{\partial\theta^2} = 0 \tag{1.3}$$

in the corresponding subsections of the ring sector. Here $\beta = 1/(\varkappa - 1)$ and $\varkappa = c_p/c_v$. Since $\tau < 1/(2\beta + 1)$, in the region under consideration Eq. (1.3) is of the elliptical type.

Following S. V. Fal'dovich [3], we will seek a solution in the form

$$\begin{aligned}
\psi_1(\tau, \theta) &= \sum_{n=1}^{\infty} [A_n z_\lambda(\tau) + B_n \zeta_\lambda(\tau)] \sin\lambda\theta, \qquad \lambda = n/\sigma \\
\psi_2(\tau, \theta) &= \frac{Q(\alpha-\theta)}{2\alpha} + \sum_{n=1}^{\infty} [C_n z_\lambda(\tau) + D_n \zeta_\lambda(\tau)] \sin\lambda\theta
\end{aligned} \tag{1.4}$$

Here the index on ψ corresponds to the number of the subsection of the ring sector in which a solution is sought; $z_\lambda(\tau)$ is the integral of the equation

$$4\tau^2(1-\tau) z_\lambda'' + 4\tau[1+(\beta-1)\tau] z_\lambda' - \lambda^2[1-(2\beta+1)\tau] z_\lambda = 0 \tag{1.5}$$

which is regular at $\tau = 0$; $\zeta_\lambda(\tau)$ is the second linear independent integral of Eq. (1.5) obtained by Lighthill [10] and Cherry [11, 12] and first used in gas-flow theory by S. V. Fal'kovich [3]. For the Wronskian of these integrals we have

$$w_\lambda(\tau) = z_\lambda'(\tau)\zeta_\lambda(\tau) - \zeta_\lambda'(\tau) z_\lambda(\tau) = \lambda(1-\tau)^\beta \tau^{-1} \tag{1.6}$$

The coefficients A_n, B_n, C_n, and D_n depend on a determination of the boundary conditions and the conditions of analytical continuation [3].

In the terms chosen the equation for gas discharge takes the form

$$Q = 2dv_1(1-\tau_1)^\beta \tag{1.7}$$

The current functions, determined by (1.4), satisfy the boundary conditions (1.1). We now use the fulfillment of the boundary conditions (1.2) as well as the conditions of analytical continuation through the boundaries of the subsection, i.e.,

$$\psi_1(\tau_1, \theta) = \psi_2(\tau_1, \theta), \quad \frac{\partial\psi_1}{\partial\tau} = \frac{\partial\psi_2}{\partial\tau} \quad \text{at} \quad \tau = \tau_1 \quad 0 \leqslant \theta \leqslant \alpha \tag{1.8}$$

The following system of equations follows from conditions (1.2) and (1.8):

$$\begin{aligned}
A_n z_\lambda(\tau_0) + B_n \zeta_\lambda(\tau_0) &= 0 \\
C_n z_\lambda(\tau_2) + D_n \zeta_\lambda(\tau_2) &= -(Q/n\pi)\cos\lambda m \\
(A_n - C_n) z_\lambda'(\tau_1) + (B_n - D_n)\zeta_\lambda'(\tau_1) &= 0 \\
(A_n - C_n) z_\lambda(\tau_1) + (B_n - D_n)\zeta_\lambda(\tau_1) &= Q/n\pi
\end{aligned}$$

Solving this system of equations and using (1.6), we determine the coefficients $A_n, \ldots, D_n$. Thus the current function (1.4) is determined, and finally the solution of the problem will take the form

$$\begin{aligned}
\psi_1(\tau, \theta) &= \frac{Q}{\alpha}\sum_{n=1}^{\infty}\frac{1}{\lambda} f_\lambda(\tau)\sin\lambda\theta \\
\psi_2(\tau, \theta) &= \frac{Q}{\alpha}\left[\frac{\alpha-\theta}{2} + \sum_{n=1}^{\infty}\frac{1}{\lambda}\chi_\lambda(\tau)\sin\lambda\theta\right]
\end{aligned} \tag{1.9}$$

Here

$$f_\lambda(\tau) = \left[\frac{T_\lambda'(\tau_1, \tau_2)}{w_\lambda(\tau_1)} - \cos\lambda m\right]\frac{T_\lambda(\tau, \tau_0)}{T_\lambda(\tau_2, \tau_0)}$$

$$\chi_\lambda(\tau) = \frac{T_\lambda'(\tau_1, \tau_0)}{w_\lambda(\tau_1)}\frac{T_\lambda(\tau, \tau_2)}{T_\lambda(\tau_2, \tau_0)} - \frac{T_\lambda(\tau, \tau_0)}{T_\lambda(\tau_2, \tau_0)}\cos\lambda m \tag{1.10}$$

where for convenience the conditions are set down [13]

$$T_\lambda(\tau, \tau_i) = z_\lambda(\tau)\zeta_\lambda(\tau_i) - z_\lambda(\tau_i)\zeta_\lambda(\tau), \quad T_\lambda(\tau_i, \tau_i) = 0$$
$$T_\lambda'(\tau_i, \tau_j) = [T_\lambda(\tau, \tau_j)]_{\tau=\tau_i}, \quad T_\lambda'(\tau_i, \tau_i) = w_\lambda(\tau_i) \quad (i, j = 0, 1, 2)$$

The characteristics of the functions $f_\lambda(\tau)$, $\chi_\lambda(\tau)$ are expressed by the following equalities [14]:

$$f_\lambda(\tau_0) = 0, \quad f_\lambda(\tau_1) - \chi_\lambda(\tau_1) = 1, \quad \chi_\lambda(\tau_2) = -\cos\lambda m$$
$$f_\lambda'(\tau_1) - \chi_\lambda'(\tau_1) = 0$$
$$f_\lambda'(\tau_0) = \left[\frac{T_\lambda'(\tau_1, \tau_2)}{w_\lambda(\tau_2)} - \cos\lambda m\right]\frac{w_\lambda(\tau_0)}{T_\lambda(\tau_2, \tau_0)} \tag{1.11}$$
$$\chi_\lambda'(\tau_2) = \frac{w_\lambda(\tau_2)}{w_\lambda(\tau_1)}\frac{T_\lambda'(\tau_1, \tau_0)}{T_\lambda(\tau_2, \tau_0)} - \frac{T_\lambda'(\tau_2, \tau_0)}{T_\lambda(\tau_2, \tau_0)}\cos\lambda m$$

$$\lim_{\tau_0\to 0} f_\lambda'(\tau_0) = 0, \quad \lim_{\tau_0\to 0}\chi_\lambda'(\tau_2) = \frac{w_\lambda(\tau_2)}{w_\lambda(\tau_1)}\frac{z_\lambda'(\tau_1)}{z_\lambda(\tau_2)} - \frac{z_\lambda'(\tau_2)}{z_\lambda(\tau_2)}\cos\lambda m \tag{1.12}$$

$$\lim_{v_m\to\infty} \tau_0 f_\lambda'(\tau_0) = \frac{\lambda}{2}\left(q_{01}{}^\lambda\frac{1+q_{12}{}^{2\lambda}}{1-q_{02}{}^{2\lambda}} - \frac{2q_{02}{}^\lambda}{1-q_{02}{}^{2\lambda}}\cos\lambda m\right)$$
$$\lim_{v_m\to\infty} \tau_2\chi_\lambda'(\tau_2) = \frac{\lambda}{2}\left(q_{12}{}^\lambda\frac{1+q_{01}{}^{2\lambda}}{1-q_{02}{}^{2\lambda}} - \frac{1+q_{02}{}^{2\lambda}}{1-q_{02}{}^{2\lambda}}\cos\lambda m\right) \tag{1.13}$$
$$q_{ij} \doteq (\tau_i/\tau_j)^{1/2} = v_i/v_j \quad (i, j = 0, 1, 2)$$

2. We determine the connection between the parameters of the problem and the dimensions of the dead air zone. Along any current line there exist general formulas

$$dx = \frac{(1-\tau)^{-\beta}}{v}\left[2\tau\frac{\partial\psi}{\partial\tau}\,d\theta - \frac{1-(2\beta+1)\tau}{2\tau(1-\tau)}\frac{\partial\psi}{\partial\theta}\,d\tau\right]\cos\theta$$
$$dy = \frac{(1-\tau)^{-\beta}}{v}\left[2\tau\frac{\partial\psi}{\partial\tau}\,d\theta - \frac{1-(2\beta+1)\tau}{2\tau(1-\tau)}\frac{\partial\psi}{\partial\theta}\,d\tau\right]\sin\theta \tag{2.1}$$

which at the jet surface τ = const are transformed to the form

$$dx = \frac{2\tau}{v(1-\tau)^\beta}\frac{\partial\psi}{\partial\tau}\cos\theta\,d\theta, \quad dy = \frac{2\tau}{v(1-\tau)^\beta}\frac{\partial\psi}{\partial\tau}\sin\theta\,d\theta \tag{2.2}$$

Substituting $\psi_1(\tau, \theta)$ from (1.9) with the calculation (1.10) into (2.2), assuming $\tau = \tau_0$ and integrating the expression obtained over θ along the jet surface LF from 0 to θ and then determining the constants of integration from the condition that $x_F = l_1\cos\alpha$, $y_L = 0$ (l_1 = OF in Fig. 1) we find the parametric equation for the dead air zone boundary LF

$$x = l_1\cos\alpha$$
$$+\frac{Q\tau_0}{\alpha}\frac{(1-\tau_0)^{-\beta}}{v_0}\sum_{n=1}^{\infty}\frac{f_\lambda'(\tau_0)}{\lambda}\left[(-1)^n\frac{2\lambda\cos\alpha}{\lambda^2-1} - \frac{\cos(\lambda+1)\theta}{\lambda+1} - \frac{\cos(\lambda-1)\theta}{\lambda-1}\right] \tag{2.3}$$

$$y = \frac{Q\tau_0}{\alpha}\frac{(1-\tau_0)^{-\beta}}{v_0}\sum_{n=1}^{\infty}\frac{f_\lambda'(\tau_0)}{\lambda}\left[\frac{\sin(\lambda-1)\theta}{\lambda-1} - \frac{\sin(\lambda+1)\theta}{\lambda+1}\right] \tag{2.4}$$

From these equalities it is easy to obtain the dimensions OF = l_1 and LO = h of the dead air zone, for which one must insert in (2.3), (2.4) the corresponding $\theta = 0$, $\theta = \alpha$, and take into account Eq. (1.7)

$$l_1 = \frac{4d}{\alpha}\left(\frac{1-\tau_1}{1-\tau_0}\right)^\beta\left(\frac{\tau_1}{\tau_0}\right)^{1/2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\lambda^2-1}\tau_0 f_\lambda'(\tau_0) \tag{2.5}$$

$$h = \frac{4d}{\alpha}\left(\frac{1-\tau_1}{1-\tau_0}\right)^{\beta}\left(\frac{\tau_1}{\tau_0}\right)^{1/2}\sum_{n=1}^{\infty}\frac{1}{\lambda^2-1}\tau_0 f_\lambda'(\tau_0) \tag{2.6}$$

An expression for the line element dl of the barrier face follows from Eq. (2.1) at $\theta = \alpha$, i.e., $d\theta = 0$

$$dl = \frac{1-(2\beta+1)\tau}{2v_m\tau^{3/2}(1-\tau)^{\beta+1}}\left(\frac{\partial\psi}{\partial\theta}\right)_{\theta=\alpha}d\tau \tag{2.7}$$

Inserting the equation for current flow (1.9) with the calculation (1.10) into Eq. (2.7), integrating the resulting expression from τ_0 to τ_2, taking into account equalities (2.5) and (1.11), and carrying out the elementary transformations analogously to [14], we obtain an expression for the segment FE = l_2 in the form

$$l_2 = \frac{d}{\sin\alpha}\left\{1-\Delta(\tau_1,\tau_2)\left[\cos m+\frac{4\sin\alpha}{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n}{\lambda^2-1}\tau_2\chi_\lambda'(\tau_2)\right]\right\}-l_1 \tag{2.8}$$

where

$$\Delta(\tau_1,\tau_2) = \left(\frac{1-\tau_1}{1-\tau_2}\right)^{\beta}\left(\frac{\tau_1}{\tau_2}\right)^{1/2}$$

Adding Eqs. (2.5) and (2.8) we obtain the length l of the barrier surface

$$l = \frac{d}{\sin\alpha}\left\{1-\Delta(\tau_1,\tau_2)\left[\cos m+\frac{4\sin\alpha}{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n}{\lambda^2-1}\tau_2\chi_\lambda'(\tau_2)\right]\right\} \tag{2.9}$$

Integrating Eq. (2.2) over θ from 0 to θ, using the expression for $\psi_2(\tau,\theta)$ from (1.9), and then taking $\tau = \tau_2$ and determining the constants of integration from the fact that the coordinates of the point B(−s, d) are known, we obtain the parametric equations of the jet contour BC

$$x = -s+\frac{Q\tau_2}{\alpha}\frac{(1-\tau_2)^{-\beta}}{v_2}\sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_2)}{\lambda}\left[\frac{2\lambda}{\lambda^2-1}-\frac{\cos(\lambda+1)\theta}{\lambda+1}-\frac{\cos(\lambda-1)\theta}{\lambda-1}\right] \tag{2.10}$$
$$y = d+\frac{Q\tau_2}{\alpha}\frac{(1-\tau_2)^{-\beta}}{v_2}\sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_2)}{\lambda}\left[\frac{\sin(\lambda-1)\theta}{\lambda-1}-\frac{\sin(\lambda+1)\theta}{\lambda+1}\right]$$

The equations for the jet contour ED are found analogously

$$x' = l\cos\alpha$$
$$+\frac{Q\tau_2}{\alpha}\frac{(1-\tau_2)^{-\beta}}{v_2}\sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_2)}{\lambda}\left[(-1)^n\frac{2\lambda\cos\alpha}{\lambda^2-1}-\frac{\cos(\lambda+1)\theta}{\lambda+1}-\frac{(\cos\lambda-1)\theta}{\lambda-1}\right] \tag{2.11}$$
$$y' = l\sin\alpha$$
$$+\frac{Q\tau_2}{\alpha}\frac{(1-\tau_2)^{-\beta}}{v_2}\sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_2)}{\lambda}\left[(-1)^n\frac{2\lambda\sin\alpha}{\lambda^2-1}+\frac{\sin(\lambda-1)\theta}{\lambda-1}-\frac{\sin(\lambda+1)\theta}{\lambda+1}\right]$$

Here x' and y' are coordinate points of the locus of contour ED.

To obtain one more relation between the parameters of the problem we take advantage of a hypothesis of N. E. Zhukovskii [15] that the points C and D lie on equipotential lines. With this assumption, following [16] and taking into account Eqs. (2.9)-(2.11) and the continuity equation

$$\delta = d\,\Delta(\tau_1,\tau_2) \tag{2.12}$$

we find a relation for $v = s/d$ in the form

$$v = -\operatorname{ctg}\alpha+\Delta(\tau_1,\tau_2)\left[\sin m+\cos m\operatorname{ctg}\alpha+\frac{4}{\alpha}\sum_{n=1}^{\infty}\frac{1}{\lambda^2-1}\tau_2\chi_\lambda'(\tau_2)\right] \tag{2.13}$$

3. We determine the resultant pressure R on the barrier by integration along its edge

$$R = 2(J + p_0 l_1 - p_2 l)\sin\alpha, \quad J = \int_{l_1}^{l} p\,dl \tag{3.1}$$

Here p is the pressure against the barrier face, p_0 is the pressure in the dead air zone, and p_2 is the pressure behind the barrier. Taking Eq. (2.8) into account and using the relations

$$p = p^\circ (1-\tau)^{\beta+1}, \quad p^\circ = v_m^{\,2} / 2(\beta+1) \tag{3.2}$$

where p^0 is the gas pressure at the stagnant point, we introduce J in the form

$$J = \frac{p^\circ}{2v_m}\left[\int_{\tau_2}^{\tau_1} \frac{1-(2\beta+1)\tau}{\tau^{3/2}} \left(\frac{\partial\psi_2}{\partial\theta}\right)_{\theta=\alpha} d\tau + \int_{\tau_1}^{\tau_0} \frac{1-(2\beta+1)\tau}{\tau^{3/2}} \left(\frac{\partial\psi_1}{\partial\theta}\right)_{\theta=\alpha} d\tau\right] \tag{3.3}$$

From Eqs. (2.5), (2.9), and (3.2), it is easy to obtain the expression

$$p_0 l_1 = \frac{2Qp^\circ}{\alpha}\frac{1-\tau_0}{v_0}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\lambda^2-1} f_\lambda{}'(\tau_0) \tag{3.4}$$

$$p_2 l = \frac{Qp^\circ}{2\sin\alpha}\frac{1-\tau_2}{v_2}\left[\Delta(\tau_2, \tau_1) - \cos m + \frac{4\sin\alpha}{\alpha}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\lambda^2-1}\tau_2 \chi_\lambda{}'(\tau_2)\right]$$

Calculating the integrals in (3.3) using (1.9) and (1.10), as was done in [14], and keeping in mind the equalities (3.2) and (3.4), we finally obtain

$$R = Qv_1 [1 + \mathcal{F}(\tau_1, \tau_2) - (\tau_2/\tau_1)^{1/2}\cos m] \tag{3.5}$$

where

$$\mathcal{F}(\tau_1, \tau_2) = \frac{1-\tau_1}{2(\beta+1)\tau_1}\left[1 - \left(\frac{1-\tau_2}{1-\tau_1}\right)^{\beta+1}\right]$$

It is easy to ascertain that at $v_m \to \infty$ the function $\mathcal{F}(\tau_1, \tau_2)$ assumes the form

$$\mathcal{F}(v_1, v_2) = (v_2^{\,2} - v_1^{\,2})/2v_1^{\,2} \tag{3.6}$$

The relations (1.9), (2.5), (2.6), (2.9), (2.13), and (3.5) provide a solution of the given problem, from which a number of published results of S. A. Chaplygin [2], L. N. Stretenskii [4], V. I. Troshin [5], E. Murgulescu [7], and N. A. Slezkin [9] follow as individual cases. For example at $\tau_0 = 0$, m = $\pi/2$, and $\sigma = 1/2 (\lambda = 2n)$ using (1.12) we obtain a solution to the problem of a gas jet emerging from a channel and striking an infinite sheet in the form

$$\psi_1(\tau, \theta) = \frac{Q}{\pi}\sum_{n=1}^{\infty}\left[(-1)^{n-1} + \frac{T'_{2n}(\tau_1, \tau_2)}{w_{2n}(\tau_1)}\right]\frac{z_{2n}(\tau)}{z_{2n}(\tau_2)}\frac{\sin 2n\theta}{n}$$

$$\psi_2(\tau, \theta) = \frac{Q}{\pi}\left\{\frac{\pi}{2} - \theta + \sum_{n=1}^{\infty}\left[(-1)^{n-1} + \frac{z'_{2n}(\tau_1)}{w_{2n}(\tau_1)}\frac{T_{2n}(\tau, \tau_2)}{z_{2n}(\tau)}\right]\frac{z_{2n}(\tau)}{z_{2n}(\tau_2)}\frac{\sin 2n\theta}{n}\right\}$$

$$\nu = \Delta(\tau_1, \tau_2)\left\{1 + \frac{8\tau_2}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}\left[(-1)^{n-1}\frac{z'_{2n}(\tau_2)}{z_{2n}(\tau_2)} + \frac{w_{2n}(\tau_2)}{w_{2n}(\tau_1)}\frac{z'_{2n}(\tau_1)}{z_{2n}(\tau_2)}\right]\right\} \tag{3.7}$$

$$R = Qv_1 [1 + \mathcal{F}(\tau_1, \tau_2)]$$

In this case R can be interpreted as the lifting force produced by infinite streams. However, in the case of a constant power input [1]

$$\tau_{2\infty}^{3/2}(1-\tau_{2\infty})^{\beta} d = \tau_2^{3/2}(1-\tau_2)^{\beta}\delta \tag{3.8}$$

a more convenient criterion of the increase in lifting force owing to proximity to the ground is the relationship of its magnitude to the thrust at a considerable distance from the ground

$$L_\infty = 2\rho_{2\infty} v_{2\infty}^2 d \tag{3.9}$$

where $v_{2\infty}$ is the discharge rate of gas from the nozzle at a considerable distance from the ground and $\rho_{2\infty}$ is the gas density in this jet.

Distributing termwise the second equation of (3.7) in Eq. (3.9) we obtain

$$c_L \equiv R / L_\infty = \Delta(\tau_1, \tau_{2\infty}) [1 + \mathcal{F}(\tau_1, \tau_2)] \tag{3.10}$$

Considering the equation of continuity (2.12), the condition of constant power input (3.8) may be represented as

$$\tau_{2\infty}^{3/2} (1 - \tau_{2\infty})^\beta = \tau_2 \tau_1^{1/2} (1 - \tau_1)^\beta \tag{3.11}$$

4. The results obtained can easily be extended to the case of an incompressible fluid. Equations (2.5), (2.6), (2.9), (2.13), and (3.5) along with condition (1.13), (3.6) take the form

$$\begin{gathered}
l_1 = \frac{2dq_{10}}{\alpha} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda}{\lambda^2 - 1} \left[q_{01}^\lambda \frac{1 + q_{12}^{2\lambda}}{1 - q_{02}^{2\lambda}} - \frac{2q_{02}^\lambda}{1 - q_{02}^{2\lambda}} \cos \lambda m \right] \\
\frac{l}{d} = \frac{1 - q_{12} \cos m}{\sin \alpha} + \frac{2q_{12}}{\alpha} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda}{\lambda^2 - 1} \left[q_{12}^\lambda \frac{1 + q_{01}^{2\lambda}}{1 - q_{02}^{2\lambda}} - \frac{1 + q_{02}^{2\lambda}}{1 - q_{02}^{2\lambda}} \cos \lambda m \right] \\
\nu = -\operatorname{ctg} \alpha + q_{12}(\sin m + \cos m \operatorname{ctg} \alpha) + \frac{2q_{12}}{\alpha} \sum_{n=1}^{\infty} \frac{\lambda}{\lambda^2 - 1} \left[q_{12}^\lambda \frac{1 + q_{01}^{2\lambda}}{1 - q_{02}^{2\lambda}} - \frac{1 + q_{02}^{2\lambda}}{1 - q_{02}^{2\lambda}} \cos \lambda m \right] \\
R = (Q / 2v_1)(v_1^2 + v_2^2 - 2v_1 v_2 \cos m)
\end{gathered} \tag{4.1}$$

The results of [6] follow from equations (4.1) at $v_1 = v_2$, and one may also obtain the well-known equations of N. E. Zhukovskii [15]. For example, supposing $\alpha = \pi/2$, $\lambda = 2n$, $v_1 = v_2$ and using the relation

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{8n \sin^2 nm}{4n^2 - 1} = \sin m \ln \operatorname{tg} \left(\frac{\pi}{4} + \frac{m}{2} \right)$$

we obtain a solution to the problem of laminar streamline flow of an incompressible fluid according to Chaplygin's system

$$\begin{gathered}
l_1 = \frac{32dq_{10}}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{4n^2 - 1} \frac{q_{01}^{2n} \sin^2 nm}{1 - q_{01}^{4n}} \\
h = \frac{32dq_{10}}{\pi} \sum_{n=1}^{\infty} \frac{n}{4n^2 - 1} \frac{q_{01}^{2n} \sin^2 nm}{1 - q_{01}^{4n}} \\
\frac{l}{d} = 2 \sin^2 \frac{m}{2} + \frac{2}{\pi} \sin m \ln \operatorname{tg} \left(\frac{\pi}{4} + \frac{m}{2} \right) + \frac{32}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{4n^2 - 1} \frac{q_{01}^{4n} \sin^2 nm}{1 - q_{01}^{4n}} \\
R = 4d\rho_1 v_1^2 \sin^2 m/2
\end{gathered} \tag{4.2}$$

Changing to the limit at $m \to 0$, $d \to \infty$ in Eq. (4.2) and taking from the equation for l the limit

$$\lim_{m \to 0} (dm^2) = \frac{2l\pi}{\pi + 4 + 64\Omega}$$

we find the well-known Chaplygin equations [8]

$$l_1 = \frac{64lq_{10}\mathrm{H}}{\pi + 4 + 64\Omega}, \quad h = \frac{64lq_{10}\Pi}{\pi + 4 + 64\Omega}, \quad R = \frac{2\rho_1 v_1^2 l\pi}{\pi + 4 + 64\Omega}$$

where

$$\begin{gathered}
H = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{4n^2 - 1} \frac{q_{01}^{2n}}{1 - q_{01}^{4n}}, \qquad \Pi = \sum_{n=1}^{\infty} \frac{n^3}{4n^2 - 1} \frac{q_{01}^{2n}}{1 - q_{01}^{4n}} \\
\Omega = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{4n^2 - 1} \frac{q_{01}^{4n}}{1 - q_{01}^{4n}}
\end{gathered}$$

TABLE 1.

	Compressible fluid				Incompressible fluid		
ν	v_1	v_2	$v_{2\infty}$	c_L	v_2	$v_{2\infty}$	c_L
0.2	24.2	213.2	104.7	1.994	199.7	98.8	2.071
0.4	24.2	103.4	63.9	1.374	102.0	63.2	1.378
0.4	54.7	259.5	158.7	1.380	230.5	142.7	1.378
0.6	24.2	70.3	49.3	1.136	70.3	49.3	1.138
0.6	54.7	163.1	114.3	1.137	158.1	110.6	1.138
0.6	76.1	244.0	171.3	1.140	221.0	154.9	1.138
0.8	24.2	53.6	41.1	1.023	55.0	41.8	1.031
0.8	54.7	124.1	94.8	1.027	124.2	94.5	1.031
0.8	76.1	180.9	138.2	1.029	172.8	131.4	1.031
0.8	107.6	278.0	213.2	1.030	244.3	185.9	1.031
1.0	24.2	45.7	36.9	0.978	46.1	37.2	0.978
1.0	54.7	104.0	84.9	0.980	104.1	84.0	0.978
1.0	76.1	146.8	119.4	0.981	144.8	116.8	0.978
1.0	107.6	214.4	175.3	0.986	204.7	165.2	0.978
1.2	24.2	39.9	33.7	0.954	40.4	34.0	0.956
1.2	54.7	89.6	75.9	0.955	91.2	76.9	0.956
1.2	76.1	128.0	108.5	0.955	126.0	106.5	0.956
1.2	107.6	181.8	155.4	0.962	179.3	151.1	0.956
1.2	152.2	268.1	232.8	0.967	253.7	214.0	0.956
1.4	24.2	35.6	31.3	0.946	36.4	31.2	0.946
1.4	54.7	80.3	70.5	0.946	82.3	71.8	0.946
1.4	76.1	114.3	100.4	0.948	114.5	99.8	0.946
1.4	107.6	162.8	143.7	0.948	161.1	140.8	0.946
1.4	152.2	234.0	209.2	0.951	229.0	200.0	0.946
1.4	186.4	293.4	267.3	0.959	280.4	244.7	0.946
1.6	24.2	33.1	29.7	0.945	33.6	30.1	0.945
1.6	54.7	74.0	66.7	0.946	75.9	65.1	0.945
1.6	76.1	105.3	94.9	0.946	105.6	94.7	0.945
1.6	107.6	143.3	131.3	0.949	149.3	133.9	0.945
1.6	152.2	212.6	194.4	0.951	211.2	181.1	0.945
1.6	186.4	259.7	241.2	0.958	258.6	231.9	0.945
1.6	215.2	298.9	282.8	0.967	298.6	267.7	0.945
1.8	24.2	30.8	28.2	0.948	31.4	28.8	0.948
1.8	54.7	69.1	63.7	0.950	71.1	65.1	0.948
1.8	76.1	98.3	90.5	0.950	99.0	90.2	0.948
1.8	107.6	137.8	127.8	0.952	139.9	128.2	0.948
1.8	152.2	194.9	182.2	0.956	197.9	181.3	0.948
1.8	186.4	238.4	225.1	0.961	242.4	222.1	0.948
1.8	215.2	275.2	262.9	0.969	279.1	255.9	0.948
2.0	24.2	29.3	27.9	0.954	29.9	27.8	0.953
2.0	54.7	65.6	61.4	0.955	67.5	62.9	0.953
2.0	76.1	92.8	87.0	0.956	93.9	87.6	0.953
2.0	107.6	131.7	123.8	0.957	132.8	123.8	0.953
2.0	152.2	183.9	174.6	0.962	187.0	174.6	0.953
2.0	186.4	223.6	214.8	0.967	230.0	214.5	0.953
2.0	215.2	257.3	248.4	0.973	265.6	247.6	0.953
2.0	240.6	280.9	274.4	0.981	296.9	276.8	0.953

Equations (3.7), (3.10), and (3.11) in the incompressible-liquid case take the form

$$\nu = q_{12} + \frac{1+q_{12}^2}{\pi}\ln\frac{1+q_{12}}{1-q_{12}}, \quad c_L = \frac{1+q_{12}^2}{2q_{12}^{2/3}}, \quad v_{2\infty} = (v_1 v_2^2)^{1/3} \tag{4.3}$$

Calculations were carried out using Eqs. (3.7), (3.10), (3.11), and (4.3) with the speed of sound a = 340 m/sec and β = 2.5, the results of which are presented in Table 1. Calculations for a compressible fluid were made according to τ, but to show the effect of compressibility the velocities are given in the table in meters per second.

The following conclusions derive from an analysis of the table and equations:

1) With an increase in ν from 0.2 to 1.6 the value of c_L, the relationship of the lifting force to the thrust at a constant power input, drops more than 45% and reaches a minimum which is less than unity. Then c_L grows with increasing ν and reaches unity for an incompressible fluid at $\nu = \infty$, for a compressible fluid at terminal velocity ν, determined by the equation

$$\nu = 1 + \frac{16\tau}{\pi}\sum_{n=1}^{\infty}{}' \frac{1}{4n^2-1}\frac{z'_{2n}(\tau)}{z_{2n}(\tau)}$$

where Σ' represents the summation over odd n;

2) the compressibility effect does not significantly increase the ground effect on the lifting force produced by vertical solid jets;

3) the additional increase in lifting force does not develop with an increase in the velocity v_1 in the case of an incompressible fluid, and this increase is not significant for compressible fluids.

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